AD-A191 050 SPATIOTEMPORAL STOCHASTIC PROCESSES(U) PRINCETON UNIV NJ DEPT OF STATISTICS G S WASTON 30 SEP 87 N00014-84-K-0421 1/1

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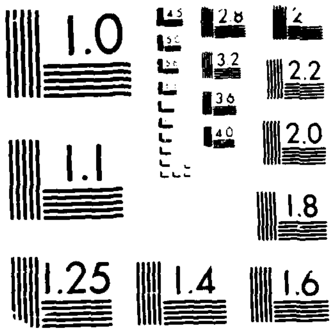

MICROCOPY RESOLUTION TEST CHART

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OFFICE OF NAVAL RESEARCH

FINAL REPORT ON ONR CONTRACT

for

1 October 1984 to 30 September 1987

for

Contract N00014-84-K-0421

TITLE: Spatiotemporal Stochastic Processes

PRINCIPAL INVESTIGATOR: Geoffrey S. Watson

Princeton University
Statistics Department
Princeton, New Jersey

CONTRACT N00014-84-K-0421

Administrative Summary

This contract was originally awarded to Watson and Antonio Possolo who left Princeton several months after it began. In the Spring of 1985 Luigi Accardi, Dept of Mathematics, University of Rome 2, came for three months to work with Watson on statistical methods in quantum mechanics. Later in August and again in September, Watson went to Europe to learn more of, and speak about, this topic. That summer Watson was supported by a NSF Grant (DMS 821381), so he received no salary from ONR. Accardi was supported almost entirely on this ONR contract from October 1, 1985 to June 30, 1986. Watson was partly paid on this contract during the summer of 1987. K.V. Mardia visited briefly in 1986 from the University of Leeds, England.

Research Summary

Watson's work with Accardi on coherent chains is described below in Accardi's survey of his research. However, the area where Watson mainly worked with Accardi on the possibilities of a theory of statistics based on quantum probability is not described. Despite an enormous investment of time and effort, and several drafts of papers, this research was abandoned because it did not interest physicists or statistical audiences at talks. This lead to the construction of interesting (classical) Markov motions on the sphere – published in a paper with Accardi and Cabrera. Watson's other papers were also concerned with distributions and inference on spheres.

Mardia's work was motivated by pattern analysis, and used Fuzzy set theory. This work was closest to the original goal of the contract.

The research of Accardi during the year 1986 can be described as follows.

Quantum Noise. The problem of constructing a mathematical model of quantum noise has a relatively short history which dates back to the early Sixties with the works of Senitzky, Schwinger, Lax, Haken, *etc*. (to quote only the names of those who can be considered pioneers of the field) but even in this short lapse of time the amount of bibliography accumulated on the subject, due to its relevance for technological applications, is impressive. Historically the physical roots of the problem were in quantum communication theory and quantum optics, especially laser theory, and in the first 15 years of development of the subject almost all the papers on the problem belong to the quantum optics literature.

The theory of classical noises has a much older history. The basic lesson one learns from it is that, prescinding from rather pathological cases, there are only two universal types of noises which can be considered as the building blocks of all the other ones: the Wiener process which is the prototype of the continuous trajectories noises, and the Poisson process which is the prototype of the purely jump (or shot) noises. All other "sufficiently regular" noises are made up of these two building blocks, in a sense which is made precise by the so-called Doob-Meyer decomposition and martingale representation theorems.

The main problem I have been investigating since 1984 is the following: Is there a class of quantum noises which can be considered the building blocks of all the "sufficiently regular" quantum noises in the same sense as the Wiener and the Poisson processes are the building blocks of all the "sufficiently regular" classical noises?

In classical probability the answer to this question passes through the devlopment of a general stochastic calculus and the subsequent use of this calculus to prove the required representation theorems. Both alone and in collaboration with other researchers, I am trying to realize this program in the context of quantum probability. In 1986 the first definite results in this direction were completed (see refs. [$a.2\cdot5\cdot3$]). Even though the paper [$a.3$] was the last to be written (for a number of contingent reasons), it was in fact chronologically the first one to introduce the crucial technical notion for the proof of the representation theorems, *i.e.*, the notion of "continuity of the trajectories for a quantum process".

(a) **Representation theorems**. The papers [$a.2\cdot5\cdot3$] contain the first quantum analogues of the classical Levy martingale representation theorem according to which to every square integrable, continuous trajectory, martingale (with a sufficiently regular conditional variance) one can associate in a canonical way a Wiener process, adapted to the filtration of the martingale, such that the martingale itself is a stochastic integral of this Wiener process.

In the paper [$a.2$] this theorem is proved, both for Bosons and Fermions, in the restricted framework of the so-called Levy fields, which allows us to avoid the delicate problem of developing a representation free quantum stochastic calculus.

In the paper [$a.5$] a more precise result is obtained, in the more restricted setting of quantum independent increment processes, as a corollary of a general theory of such processes.

Finally in the paper [$a.3$] the result is established in full generality (even for a multidimensional index set), but only for the Fermion case where one can take advantage of the boundedness of the operators involved.

The proof for the general Boson case is more difficult since it presupposes the development of a complete theory of representation independent stochastic integration. This proof has been completed only recently in collaboration with Fagnola and Quaegebeur. The following section gives more information on this paper.

b.) Quantum stochastic integration

In order to prove the quantum Levy martingale representation theorem in full generality a general theory of quantum stochastic integration must be constructed . Several examples of quantum stochastic integration have been studied in the framework of some very special Gaussian representation of the CCR or of the CAR however, since in all these examples the representation (i.e. the model of quantum noise) is fixed a priori, they cannot in principle be of any help for the solution of our problem which, as stated above , consists in the classification of all the possible models of quantum noises. An outline of this general theory is contained in [g.3],a first precise tretment of the stochastic integration without the Ito formula is in [d.3] while the basic ingredient for the proof of the Ito formula, i.e the notion of mutual quadratic variation (square brackets, crochet droit) of two processes, is developed in [a.4] where the computation of the brackets of all the basic integrators is carried out explicitly in the case of Gaussian representations. A paper in which all these different pieces should be put together to prove the first existence uniqueness and unitarity result for a quantum stochastic differential equation in a representation free context is in preparation by Accardi, Fagnola and Quaegebeur and should be ready in the first months of '88.

c.) Properties of the quantum noises

A number of interesting questions could be studied independently on the classification of the basic quantum noises for example :

(I.) is it possible to describe in a qualitative, model independent , way those quantum systems in which the appearence of a kind of quantum noise should be reasonably expected ?

(II.) how should the Schroedinger equation be modified for such systems ?

(III.) are there some universal relations among the the parameters entering in the equations describing noisy quantum systems , i.e. some relations which depend only on the basic principles of quantum theory and not on some specific model ?

(IV.) assuming that one can find an answer to question (II.), how are these new equations related to the considerable number of phenomenological equations devised in the physical literature to describe dissipative systems ?

The answers to these questions are in my opinion the main result I obtained during my stay in Princeton. They are contained in the paper [a.1] a preliminary version of which is [g.1]. It turns out that :

- starting from the basic principles of quantum theory one can define a hierarchy of chaoticity for quantum systems which generalizes the corresponding hierarchy for classical stationary processes.

- for a certain class of chaotic quantum systems the Scroedinger equation (in interaction representation) has a natural generalization into a stochastic Scroedinger equation (answer to question (II.)) from which one can deduce a quantum generalization of the Einstein fluctuation-dissipation relation connecting the covariance of the noise (fluctuation) with the drift (dissipation) in the classical Langevin equation (answer to question (III.)).

- the above equation induces on the observables a very general form of quantum Langevin equation . When particularised to observables that commute with the noise these equations give a model independent (at least in the class singled out by the chaoticity assumptions) extension of all the known equations introduced in the physical literature to describe dissipative quantum systems. In particular we obtain a generalization of the Senitzky-Lax equations introduced in the early sixties to describe the dumped harmonic oscillator . Much more interesting is the generalization that we obtain of the Bloch equations, introduced in 1946 to describe nuclear magnetic resonance (i.e. the absorption of EM radiation from the atomic nuclei), in fact in this case our theory predicts a nonlinear correction to these equations . The theory also explains why Bloch did not include this nonlinear correction : the reason is that the correction is bilinear in the generators of the rotation group and it is well known that in the spin-1/2 case -the only one considered by Bloch - the product of two Pauli matrices is still a Pauli matrix . In fact in the spin-1/2 case the nonlinear equations predicted by our theory reduce exactly to the usual linear Bloch equations .

Thus our theory not only unifies several phenomenological equations, obtained in different contexts at different times, but also predicts experimentally measurable corrections to such equations and provides a generalization of these equations to an arbitrary Lie group.

In collaboration with M.Abundo I am now performing numerical experiments in the 3×3 case to check the discrepancy of predictions in the linear and nonlinear case. On the coefficients of the time evolved matrix this discrepancy can be of the order of 20% , but at the moment we have not yet the complete results on the physical parameters, which are functions of these coefficients.

d.) Quantum independent increment processes.

The classification of classical independent increment processes is reduced to the classification of convolution semigroups. Underlying the notion of convolution there is always that of comultiplication (on the coalgebra of the coefficients of the finite dimensional representations of the group with the natural operations). Therefore the natural framework to extend the classical theory of independent increment processes is that of coalgebras. The main results of the paper [a.5] are :

- the construction of a C^*-algebra canonically associated to a convolution semigroup (or more generally evolution) on a coassociative *-bialgebra . To my knowledge this is the first example of a continuous tensor product of C^*-algebras which does not come from a Gaussian representation of the CCR.

- the proof of the quantum Levy-Khintchin formula, which when particularized to the classical case provides a new proof of this known result.

- the above mentioned proof , in this context, of the quantum Levy theorem.

Quantum Markov chains

In collaboration with G.S.Watson [b.1] the theory of quantum Markov chains has been applied to construct a class of explicitly solvable nontrivial (i.e. non free) one dimensional quantum lattice systems with an infinite dimensional state space. The infinite dimensionality is crucial for the non triviality of the result, since in this case the known results on quantum spin systems do not apply by lack of compactness . In particular we have produced a simple constructive example of perturbation of the free Gibbs state on an infinite one dimensional lattice which is a simplified version of the known Hubbard model (simplified since the current term is missing) . The general technique described in the first section of the paper enables one to include also the current term, but we did not do that, since at the time we were not aware of the existence of the Hubbard model.

FOUNDATIONS OF QUANTUM MECHANICS

The paper [d.1] is the first systematic attempt to describe the quantum probabilistic approach to the foundations of quantum theory. In it I included several unpublished results that I obtained in previous years on this topic, while I did not include the results I had already published elsewhere.

The note [d.2] is a technical Lemma on von Neuman' s canonical form for a state of a composite quantum system where, completing von Neumann' s analysis, the role of degeneracy is taken explicitly into account . The existence of an analogue decomposition for general states on a von Neumann algebra is still an open problem.

Publications & Research Activity

a.) Papers submitted for publication to refereed journals:

[1] L. Accardi, "A mathematical theory of quantum noise," (1987). Submitted to *Phys. Rev. A*.

[2] L. Accardi & K. R. Parthasarathy, "A martingale characterization of canonical commutation and anticommutation relations," (1986). Submitted to *Journ. Funct. Anal.*.

[3] L. Accardi & J. Quaegebeur, "The Fermion Levy martingale representation theorem." (1987) Submitted to *Ann. Ist. Fourier*

[4] L. Accardi & J. Quaegebeur, "Ito algebras of Gaussian quantum fields," (1988). Submitted to *Journ. Funct. Anal.*.

[5] L. Accardi, M. Schurmann & W. von Waldenfels, "Quantum independent increment processes on superalgebras," (1987). Submitted to *Math. Zeitschr.*.

[6] L. Accardi & G. S. Watson, "Quantum random walks and coherent quantum chains," (1987). Submitted to *Quantum Probability and Applications* **IV**, Springer, LNM.

[7] L. Accardi, G. S. Watson & J. Cabrera, "Markov motions on the sphere," (1987). Submitted to *Metron*

[8] J. T. Kent & K. V. Mardia, "Fuzzy classification in signal processing," (1985). To appear in *Proceedings of Inter. Conf. on Maths. in Signal Processing, IMA*.

[9] J. T. Kent & K. V. Mardia, "Spatial classification using fuzzy membership models," (1985). To appear in *IEEE Trans. on Pattern Analysis and Machine Intelligence*.

[10] K. V. Mardia, "Multi-dimensional multiariate Gaussian Markov random fields," (1987). To appear in *Journal of Multivariate Analysis*.

[11] G. S. Watson, "Permutation tests for the independence of a scalar and a vector," (1987) Submitted to *Aust. J. Stats.*

[12] G. S. Watson, "The Langevin distribution on high-dimensional spheres," (1987) Submitted to *J. App. Stats.*

b.) Papers published in refereed journals:

[1] L. Accardi & G. S. Watson, "Markov states of the electromagnetic field," in *Phys. Rev. Letters* **35**, (1987),pp. 1275-83.

c.) Books (or sections thereof) submitted for publication:

[1] L. Accardi, F. Fagnola & J. Quaegebeur, "Quantum Stochastic Calculus." To appear

d.) Books (or sections thereof) published:

[1] L. Accardi, "Foundations of quantum mechanics: A quantum probabilistic approach," *Quantum Paradoxes*, Reidel, (1987).

[2] L. Accardi, "On the universality of the Einstein-Podolsky-Rosen phenomenon," *Information and Control in Quantum Physics*, Springer, (1987).

[3] L. Accardi & F. Fagnola, "Stochastic integration," *Quantum Probability and Applications, III*, (1987), Springer.

g.) Conference presentations:

[1] L. Accardi, "Mathematical theory of quantum noise," *Proceedings on the 1st World Conference of the Bernoulli Society*, Tashkent, (1986), VNU Science Press.

[2] L. Accardi, "Non kolmogorovian probabilistic models and quantum theory," *Proceedings of the 45th ISI Conference*, Amsterdam, (1985).

[3] L. Accardi, "Quantum stochastic calculus," *Proceedings of the 4th Vilnius Conference on Probability and Mathematical Statistics*, (1986), VNU Science Press.

[4] K. V. Mardia, "Statistics of directional data: An overview," presented to ISI by Dr. P. Jupp, (1987).

[5] G. S. Watson, "Probability in physics," *Proceedings of the 45th ISI Conference*, Amsterdam, (1985).

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